

# **Cass Technical High School**

## **Class of 2019 Summer Math Work**

The purpose of this summer packet is to review the foundational concepts needed to ensure that you are successful in your math class for the 2016-2017 school year.

You are responsible for printing your packet which can be found on the Cass Tech's website, <http://casstech.schools.detroitk12.org/>.

**PACKETS ARE DUE TUESDAY, SEPTEMBER 6, 2016 AND MUST BE SUBMITTED TO YOUR MATH TEACHER. LATE PACKETS WILL NOT BE ACCEPTED.**

You must show all work, if there is not enough space on the assignment please use additional sheets of paper. Please record answers on the answer sheet provided. In the event that you are struggling with a concept it is highly recommended that you seek tutorial services from a personal tutor or an online resource such as [www.khanacademy.org](http://www.khanacademy.org).

This packet is very lengthy, thus it is highly recommended that you work on this work throughout the summer.

Please contact Ms. Patricia Perry, [patricia.perry@detroitk12.org](mailto:patricia.perry@detroitk12.org) if you have any questions.

# TOPIC 1: GRAPHING ORDERED PAIRS

Points in the coordinate plane are named by **ordered pairs** of the form  $(x, y)$ .  
 The first number, or  **$x$ -coordinate**, corresponds to a number on the  $x$ -axis.  
 The second number, or  **$y$ -coordinate**, corresponds to a number on the  $y$ -axis.

## EXAMPLE

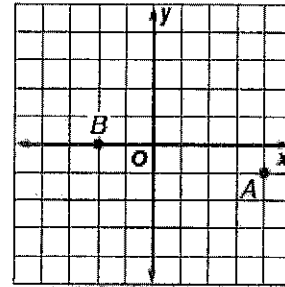
1 Write the ordered pair for each point.

a. A

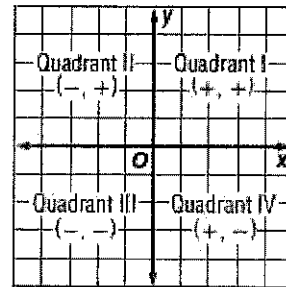
The  $x$ -coordinate is 4.  
 The  $y$ -coordinate is  $-1$ .  
 The ordered pair is  $(4, -1)$ .

b. B

The  $x$ -coordinate is  $-2$ .  
 The point lies on the  $x$ -axis,  
 so its  $y$ -coordinate is 0.  
 The ordered pair is  $(-2, 0)$ .



The  $x$ -axis and  $y$ -axis separate the coordinate plane into four regions, called **quadrants**. The point at which the axes intersect is called the **origin**. The axes and points on the axes are not located in any of the quadrants.



4 Graph four points that satisfy the equation  $y = 4 - x$ .

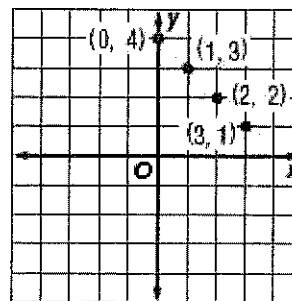
Make a table.

Choose four values for  $x$ .

Evaluate each value of  $x$  for  $4 - x$ .

$x$	$4 - x$	$y$	$(x, y)$
0	$4 - 0$	4	$(0, 4)$
1	$4 - 1$	3	$(1, 3)$
2	$4 - 2$	2	$(2, 2)$
3	$4 - 3$	1	$(3, 1)$

Plot the points.



## PRACTICE PROBLEMS

Graph 4 points that satisfy the equation.

1.  $y = 2x$

2.  $3x - y = 1$

3.  $5x + 2y = -4$

## TOPIC 2: EVALUATING ALGEBRAIC EXPRESSIONS

An expression is an algebraic expression if it contains sums and/or products of variables and numbers. To evaluate an algebraic expression, replace the variable or variables with known values, and then use the order of operations.

### PEMDAS

1<sup>st</sup> - Parentheses

2<sup>nd</sup> - Exponents

3<sup>rd</sup> - Multiplication/Division

4<sup>th</sup> - Addition/Subtraction

### EXAMPLE

1 Evaluate each expression.

a.  $x - 5 + y$  if  $x = 15$  and  $y = -7$

$$\begin{aligned}x - 5 + y &= 15 - 5 + (-7) && \text{Substitute.} \\ &= 10 + (-7) && \text{Subtract.} \\ &= 3 && \text{Add.}\end{aligned}$$

b.  $6ab^2$  if  $a = -3$  and  $b = 3$

$$\begin{aligned}6ab^2 &= 6(-3)(3)^2 && \text{Substitute.} \\ &= 6(-3)(9) && 3^2 = 9 \\ &= (-18)(9) && \text{Multiply.} \\ &= -162 && \text{Multiply.}\end{aligned}$$

### EXAMPLE

2 Evaluate if  $m = -2$ ,  $n = -4$ , and  $p = 5$ .

a.  $\frac{2m + n}{p - 3}$

$$\begin{aligned}\frac{2m + n}{p - 3} &= \frac{2(-2) + (-4)}{5 - 3} && \text{Substitute.} \\ &= \frac{-4 - 4}{5 - 3} && \text{Multiply.} \\ &= \frac{-8}{2} \text{ or } -4 && \text{Subtract.}\end{aligned}$$

b.  $-3(m^2 + 2n)$

$$\begin{aligned}-3(m^2 + 2n) &= -3[(-2)^2 + 2(-4)] \\ &= -3[4 + (-8)] \\ &= -3(-4) \text{ or } 12\end{aligned}$$

## PRACTICE PROBLEMS

Evaluate each expression if  $a = 2$ ,  $b = -3$ ,  $c = -1$ , and  $d = 4$

4.  $\frac{3b}{5a + 2}$

5.  $\frac{bd}{2c}$

6.  $\frac{2d - a}{b}$

7.  $5 + d(3b - 2d)$

8.  $-2(b^2 - 5c)$

## TOPIC 3: SOLVING EQUATIONS

If the same number is added to or subtracted from each side of an equation, the resulting equation is true.

### EXAMPLE

1 Solve each equation.

a.  $x - 7 = 16$

$$x - 7 = 16$$

Original equation

$$x - 7 + 7 = 16 + 7$$

Add 7 to each side.

$$x = 23$$

Simplify.

b.  $m + 12 = -5$

$$m + 12 = -5$$

Original equation

$$m + 12 + (-12) = -5 + (-12)$$

Add  $-12$  to each side.

$$m = -17$$

Simplify.

c.  $k + 31 = 10$

$$k + 31 = 10$$

Original equation

$$k + 31 - 31 = 10 - 31$$

Subtract 31 from each side.

$$k = -21$$

Simplify.

2 Solve each equation.

a.  $4d = 36$

$$4d = 36$$

Original equation

$$\frac{4d}{4} = \frac{36}{4}$$

Divide each side by 4.

$$d = 9$$

Simplify.

b.  $-\frac{t}{8} = -7$

$$-\frac{t}{8} = -7$$

Original equation

$$-8\left(-\frac{t}{8}\right) = -8(-7)$$

Multiply each side by  $-8$ .

$$t = 56$$

Simplify.

c.  $\frac{3}{5}x = -8$

$$\frac{3}{5}x = -8$$

Original equation

$$\frac{5}{3}\left(\frac{3}{5}\right)x = \frac{5}{3}(-8)$$

Multiply each side by  $\frac{5}{3}$ .

$$x = -\frac{40}{3}$$

Simplify.

To solve equations with more than one operation, often called *multi-step equations*, undo operations by working backward.

**3** Solve each equation.

a.  $8q - 15 = 49$

$$8q - 15 = 49$$

Original equation

$$8q = 64$$

Add 15 to each side.

$$q = 8$$

Divide each side by 8.

b.  $12y + 8 = 6y - 5$

$$12y + 8 = 6y - 5$$

Original equation

$$12y = 6y - 13$$

Subtract 8 from each side.

$$6y = -13$$

Subtract  $6y$  from each side.

$$y = -\frac{13}{6}$$

Divide each side by 6.

**4** Solve  $3(x - 5) = 13$ .

$$3(x - 5) = 13$$

Original equation

$$3x - 15 = 13$$

Distributive Property

$$3x = 28$$

Add 15 to each side.

$$x = \frac{28}{3}$$

Divide each side by 3.

## PRACTICE PROBLEMS

Solve each equation. Express final answer in fraction form.

9.  $\frac{8}{5}a = -6$

10.  $-\frac{p}{12} = 8$

11.  $5c - 7 = 8c - 4$

12.  $\frac{7}{4}x - 2 = -5x + 1$

13.  $-3(d - 7) = 6 - 5d$

14.  $\frac{5}{x} + 4 = 12$

15.  $\frac{3}{x+2} = 4$

16.  $\frac{3}{2}x + 5 = \frac{2}{3}x - 10$

## TOPIC 4: SOLVING INEQUALITIES

Statements with greater than ( $>$ ), less than ( $<$ ), greater than or equal to ( $\geq$ ), or less than or equal to ( $\leq$ ) are inequalities.

If any number is added or subtracted to each side of an inequality, the resulting inequality is true.

### EXAMPLE

**1** Solve each inequality.

a.  $x - 17 > 12$

$$x - 17 > 12 \quad \text{Original inequality}$$

$$x - 17 + 17 > 12 + 17 \quad \text{Add 17 to each side.}$$

$$x > 29 \quad \text{Simplify.}$$

The solution set is  $\{x \mid x > 29\}$ .

b.  $y + 11 \leq 5$

$$y + 11 \leq 5 \quad \text{Original inequality}$$

$$y + 11 - 11 \leq 5 - 11 \quad \text{Subtract 11 from each side.}$$

$$y \leq -6 \quad \text{Simplify.}$$

The solution set is  $\{y \mid y \leq -6\}$ .

If each side of an inequality is multiplied or divided by a positive number, the resulting inequality is true.

**2** Solve each inequality.

a.  $\frac{t}{6} \geq 11$

$$\frac{t}{6} \geq 11 \quad \text{Original inequality}$$

$$(6)\frac{t}{6} \geq (6)11 \quad \text{Multiply each side by 6.}$$

$$t \geq 66 \quad \text{Simplify.}$$

The solution set is  $\{t \mid t \geq 66\}$ .

b.  $8p < 72$

$$8p < 72 \quad \text{Original inequality}$$

$$\frac{8p}{8} < \frac{72}{8} \quad \text{Divide each side by 8.}$$

$$p < 9 \quad \text{Simplify.}$$

The solution set is  $\{p \mid p < 9\}$ .

If each side of an inequality is multiplied or divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is true.

**3** Solve each inequality.

a.  $-5c > 30$

$$-5c > 30 \quad \text{Original inequality}$$

$$\frac{-5c}{-5} < \frac{30}{-5} \quad \text{Divide each side by } -5. \text{ Change } > \text{ to } <.$$

$$c < -6 \quad \text{Simplify.}$$

The solution set is  $\{c \mid c < -6\}$ .

b.  $-\frac{d}{13} \leq -4$

$-\frac{d}{13} \leq -4$  Original inequality

$(-13)\left(\frac{-d}{13}\right) \geq (-13)(-4)$  Multiply each side by  $-13$ . Change  $\leq$  to  $\geq$ .  
 $d \geq 52$  Simplify.

The solution set is  $\{d | d \geq 52\}$ .

**4** Solve each inequality.

a.  $-6a + 13 < -7$

$-6a + 13 < -7$  Original inequality

$-6a + 13 - 13 < -7 - 13$  Subtract 13 from each side.

$-6a < -20$  Simplify.

$\frac{-6a}{-6} > \frac{-20}{-6}$  Divide each side by  $-6$ . Change  $<$  to  $>$ .

$a > \frac{10}{3}$  Simplify.

The solution set is  $\left\{a \mid a > \frac{10}{3}\right\}$ .

b.  $4z + 7 \geq 8z - 1$

$4z + 7 \geq 8z - 1$  Original inequality

$4z + 7 - 7 \geq 8z - 1 - 7$  Subtract 7 from each side.

$4z \geq 8z - 8$  Simplify.

$4z - 8z \geq 8z - 8 - 8z$  Subtract  $8z$  from each side.

$-4z \geq -8$  Simplify.

$\frac{-4z}{-4} \leq \frac{-8}{-4}$  Divide each side by  $-4$ . Change  $\geq$  to  $\leq$ .

$z \leq 2$  Simplify.

The solution set is  $\{z | z \leq 2\}$ .

## PRACTICE PROBLEMS

Solve each inequality.

17.  $-\frac{a}{8} < 5$

18.  $-3n - 8 > 2n + 7$

19.  $-\frac{4}{5}k - 17 \geq 11$

20.  $\frac{2}{3}x + 12 \leq \frac{1}{4}x - 3$

21.  $3(2x + 6) - 3 < -2(x + 5)$

# TOPIC 5: GRAPHING EQUATIONS

## EXAMPLE

- 1 Determine the  $x$ -intercept and  $y$ -intercept of  $4x - 3y = 12$ . Then graph the equation.

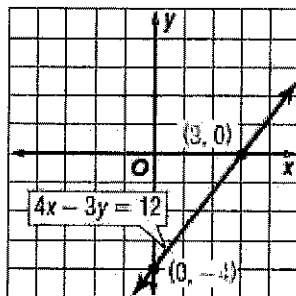
To find the  $x$ -intercept, let  $y = 0$ .

$$\begin{aligned}4x - 3y &= 12 && \text{Original equation} \\4x - 3(0) &= 12 && \text{Replace } y \text{ with } 0. \\4x &= 12 && \text{Simplify.} \\x &= 3 && \text{Divide each side by } 4.\end{aligned}$$

To find the  $y$ -intercept, let  $x = 0$ .

$$\begin{aligned}4x - 3y &= 12 && \text{Original equation} \\4(0) - 3y &= 12 && \text{Replace } x \text{ with } 0. \\-3y &= 12 && \text{Divide each side by } -3. \\y &= -4 && \text{Simplify.}\end{aligned}$$

Put a point on the  $x$ -axis at 3 and a point on the  $y$ -axis at  $-4$ . Draw the line through the two points.



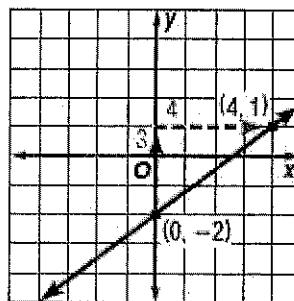
A linear equation of the form  $y = mx + b$  is in *slope-intercept* form, where  $m$  is the slope and  $b$  is the  $y$ -intercept.

- 2 Graph  $y = \frac{3}{4}x - 2$ .

**Step 1** The  $y$ -intercept is  $-2$ . So, plot a point at  $(0, -2)$ .

**Step 2** The slope is  $\frac{3}{4}$ .  $\frac{\text{rise}}{\text{run}}$   
From  $(0, -2)$ , move up 3 units and right 4 units. Plot a point.

**Step 3** Draw a line connecting the points.



## PRACTICE PROBLEMS

Graph each equation using  $x$ - and  $y$ -intercepts.

22.  $-2x + 3y = 6$

23.  $3x - y = 3$

Graph each equation using slope and  $y$ -intercept.

24.  $y = x + 4$

25.  $3x - 2y = 12$

Graph each equation using either method.

26.  $y = \frac{2}{3}x - 3$

27.  $-6x + y = 2$

28.  $3x + 4y = -12$



## TOPIC 6: SQUARE ROOTS AND SIMPLIFYING RADICALS

A radical expression is an expression that contains a square root. The expression is in simplest form when the following three conditions have been met.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

The **Product Property** states that for two numbers  $a$  and  $b \geq 0$ ,  $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ .

### EXAMPLE

**1** Simplify.

a.  $\sqrt{45}$

$$\begin{aligned}\sqrt{45} &= \sqrt{3 \cdot 3 \cdot 5} \\ &= \sqrt{3^2} \cdot \sqrt{5} \\ &= 3\sqrt{5}\end{aligned}$$

Prime factorization of 45

Product Property of Square Roots

Simplify.

b.  $\sqrt{6} \cdot \sqrt{15}$

$$\begin{aligned}\sqrt{6} \cdot \sqrt{15} &= \sqrt{6 \cdot 15} \\ &= \sqrt{3 \cdot 2 \cdot 3 \cdot 5} \\ &= \sqrt{3^2} \cdot \sqrt{10} \\ &= 3\sqrt{10}\end{aligned}$$

Product Property

Prime factorization

Product Property

Simplify.

For radical expressions in which the exponent of the variable inside the radical is *even* and the resulting simplified exponent is *odd*, you must use absolute value to ensure nonnegative results.

The **Quotient Property** states that for any numbers  $a$  and  $b$ , where  $a \geq 0$  and

$$b \geq 0, \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

### EXAMPLE

**3** Simplify  $\sqrt{\frac{25}{16}}$ .

$$\begin{aligned}\sqrt{\frac{25}{16}} &= \frac{\sqrt{25}}{\sqrt{16}} && \text{Quotient Property} \\ &= \frac{5}{4} && \text{Simplify.}\end{aligned}$$

Rationalizing the denominator of a radical expression is a method used to eliminate radicals from the denominator of a fraction. To rationalize the denominator, multiply the expression by a fraction equivalent to 1 such that the resulting denominator is a perfect square.

### EXAMPLE

4 Simplify.

a.  $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad \text{Multiply by } \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3} \quad \text{Simplify.}$$

b.  $\frac{\sqrt{13y}}{\sqrt{18}}$

$$\frac{\sqrt{13y}}{\sqrt{18}} = \frac{\sqrt{13y}}{\sqrt{2 \cdot 3 \cdot 3}} \quad \text{Prime factorization}$$

$$= \frac{\sqrt{13y}}{3\sqrt{2}} \quad \text{Product Property}$$

$$= \frac{\sqrt{13y}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{Multiply by } \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{26y}}{6} \quad \text{Product Property}$$

Sometimes, conjugates are used to simplify radical expressions. Conjugates are binomials of the form  $p\sqrt{q} + r\sqrt{s}$  and  $p\sqrt{q} - r\sqrt{s}$ .

### PRACTICE PROBLEMS

SIMPLIFY.

29.  $\sqrt{32}$

30.  $\sqrt{75}$

31.  $\sqrt{147}$

32.  $\sqrt{50} \cdot \sqrt{10}$

33.  $4\sqrt{12} \cdot 2\sqrt{3}$

34.  $\sqrt{\frac{18}{36}}$

35.  $\frac{3}{\sqrt{48}}$

36.  $\frac{2\sqrt{5}}{\sqrt{24}}$

## TOPIC 7: MULTIPLYING POLYNOMIALS

The **Product of Powers** rule states that for any number  $a$  and all integers  $m$  and  $n$ ,  
 $a^m \cdot a^n = a^{m+n}$ .

### EXAMPLE

**1** Simplify each expression.

a.  $(4p^5)(p^4)$

$$\begin{aligned}(4p^5)(p^4) &= (4)(1)(p^5 \cdot p^4) \\ &= (4)(1)(p^{5+4}) \\ &= 4p^9\end{aligned}$$

b.  $(3yz^5)(-9y^2z^2)$

$$\begin{aligned}(3yz^5)(-9y^2z^2) &= (3)(-9)(y \cdot y^2)(z^5 \cdot z^2) \\ &= -27(y^{1+2})(z^{5+2}) \\ &= -27y^3z^7\end{aligned}$$

**2** Simplify  $3x^3(-4x^2 + x - 5)$ .

$$\begin{aligned}3x^3(-4x^2 + x - 5) &= 3x^3(-4x^2) + 3x^3(x) - 3x^3(5) && \text{Distributive Property} \\ &= -12x^5 + 3x^4 - 15x^3 && \text{Multiply.}\end{aligned}$$

To find the power of a power, multiply the exponents. This is called the **Power of a Power** rule.

**3** Simplify each expression.

a.  $(-3x^2y^4)^3$

$$\begin{aligned}(-3x^2y^4)^3 &= (-3)^3(x^2)^3(y^4)^3 \\ &= -27x^6y^{12}\end{aligned}$$

b.  $(xy)^3(-2x^4)^2$

$$\begin{aligned}(xy)^3(-2x^4)^2 &= x^3y^3(-2)^2(x^4)^2 \\ &= x^3y^3(4)x^8 \\ &= 4x^3 \cdot x^8 \cdot y^3 \\ &= 4x^{11}y^3\end{aligned}$$

To multiply two binomials, find the sum of the products of

- F the *First* terms,
- O the *Outer* terms,
- I the *Inner* terms, and
- L the *Last* terms.

**4** Find  $(2x - 3)(x + 1)$ .

$$\begin{aligned}(2x - 3)(x + 1) &= (2x)(x) + (2x)(1) + (-3)(x) + (-3)(1) && \text{FOIL method} \\ &= 2x^2 + 2x - 3x - 3 && \text{Multiply.} \\ &= 2x^2 - x - 3 && \text{Combine like terms.}\end{aligned}$$

**5** Find  $(3x - 2)(2x^2 + 7x - 4)$ .

$$\begin{aligned}(3x - 2)(2x^2 + 7x - 4) &= 3x(2x^2 + 7x - 4) - 2(2x^2 + 7x - 4) && \text{Distributive Property} \\ &= 6x^3 + 21x^2 - 12x - 4x^2 - 14x + 8 && \text{Distributive Property} \\ &= 6x^3 + 17x^2 - 26x + 8 && \text{Combine like terms.}\end{aligned}$$

Three special products are  $(a + b)^2 = a^2 + 2ab + b^2$ ,  
 $(a - b)^2 = a^2 - 2ab + b^2$ , and  
 $(a + b)(a - b) = a^2 - b^2$ .

**6** Find each product.

a.  $(2x - z)^2$

$$\begin{aligned}(a - b)^2 &= a^2 - 2ab + b^2 && \text{Square of a difference} \\ (2x - z)^2 &= (2x)^2 - 2(2x)(z) + (z)^2 && a = 2x \text{ and } b = z \\ &= 4x^2 - 4xz + z^2 && \text{Simplify.}\end{aligned}$$

b.  $(3x + 7)(3x - 7)$

$$\begin{aligned}(a + b)(a - b) &= a^2 - b^2 && \text{Product of sum and difference} \\ (3x + 7)(3x - 7) &= (3x)^2 - (7)^2 && a = 3x \text{ and } b = 7 \\ &= 9x^2 - 49 && \text{Simplify.}\end{aligned}$$

## PRACTICE PROBLEMS

Find each product.

37.  $\left(\frac{8}{5}x^3y\right)(4x^3y^2)$

38.  $4m^2(-2m^2 + 7m - 5)$

39.  $(-5w^3x^2)^2(2w^5)^3$

40.  $(a + 3)(a - 6)$

41.  $(5d + 3)(2d - 4)$

42.  $(4a - 3)(4a + 3)$

43.  $(x - 5)^2$

44.  $(3x + 2)^2$

45.  $(x - 2)(x^2 + 3x - 7)$

## TOPIC 8: FACTORING TO SOLVE EQUATIONS

Some polynomials can be factored using the Distributive Property.

### EXAMPLE

#### 1 Factor $5t^2 + 15t$ .

Find the greatest common factor (GCF) of  $5t^2$  and  $15t$ .

$$5t^2 = 5 \cdot t \cdot t, 15t = 3 \cdot 5 \cdot t \quad \text{GCF: } 5 \cdot t \text{ or } 5t$$

$$\begin{aligned} 5t^2 + 15t &= 5t(t) + 5t(3) && \text{Rewrite each term using the GCF.} \\ &= 5t(t + 3) && \text{Distributive Property} \end{aligned}$$

To factor polynomials of the form  $x^2 + bx + c$ , find two integers  $m$  and  $n$  so that  $mn = c$  and  $m + n = b$ . Then write  $x^2 + bx + c$  using the pattern  $(x + m)(x + n)$ .

To factor polynomials of the form  $ax^2 + bx + c$ , find two integers  $m$  and  $n$  with a product equal to  $ac$  and with a sum equal to  $b$ . Write  $ax^2 + bx + c$  using the pattern  $ax^2 + mx + nx + c$ . Then factor by grouping.

#### 2 Factor each polynomial.

a.  $x^2 - 8x + 15$

In this equation,  $b$  is  $-8$  and  $c$  is  $15$ . This means that  $m + n$  is negative and  $mn$  is positive. So  $m$  and  $n$  must both be negative.

$$\begin{aligned} x^2 - 8x + 15 &= (x + m)(x + n) \\ &= (x - 3)(x - 5) \end{aligned}$$

$b$  is negative and  $c$  is positive.

Factors of 15	Sum of Factors
$-1, -15$	$-16$
$-3, -5$	$-8$

The correct factors are  $-3$  and  $-5$ .

Write the pattern;  $m = -3$  and  $n = -5$

b.  $5x^2 - 19x - 4$

In this equation,  $a$  is  $5$ ,  $b$  is  $-19$ , and  $c$  is  $-4$ . Find two numbers with a product of  $-20$  and with a sum of  $-19$ .

$$\begin{aligned} 5x^2 - 19x - 4 &= 5x^2 + mx + nx - 4 \\ &= 5x^2 + x + (-20)x - 4 \\ &= (5x^2 + x) - (20x + 4) \\ &= x(5x + 1) - 4(5x + 1) \\ &= (x - 4)(5x + 1) \end{aligned}$$

$b$  is negative and  $c$  is negative.

Factors of $-20$	Sum of Factors
$-2, 10$	$8$
$2, -10$	$-8$
$-1, 20$	$19$
$1, -20$	$-19$

Factor the GCF from each group.

Distributive Property

Here are some special products.

#### Perfect Square Trinomials

$$\begin{aligned} a^2 + 2ab + b^2 &= (a + b)(a + b) \\ &= (a + b)^2 \end{aligned}$$

$$\begin{aligned} a^2 - 2ab + b^2 &= (a - b)(a - b) \\ &= (a - b)^2 \end{aligned}$$

#### Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

**3** Factor each polynomial.

a.  $9x^2 + 6x + 1$

The first and last terms are perfect squares, and the middle term is equal to  $2(3x)(1)$ .

$$\begin{aligned} 9x^2 + 6x + 1 &= (3x)^2 + 2(3x)(1) + 1^2 && \text{Write as } a^2 + 2ab + b^2. \\ &= (3x + 1)^2 && \text{Factor using the pattern.} \end{aligned}$$

b.  $x^2 - 9 = 0$

This is a difference of squares.

$$\begin{aligned} x^2 - 9 &= x^2 - (3)^2 && \text{Write in the form } a^2 - b^2. \\ &= (x - 3)(x + 3) && \text{Factor the difference of squares.} \end{aligned}$$

The binomial  $x - a$  is a factor of the polynomial  $f(x)$  if and only if  $f(a) = 0$ . Since 0 times any number is equal to zero, this implies that we can use factoring to solve equations.

**4** Solve  $x^2 - 5x + 4 = 0$  by factoring.

$$\begin{aligned} x^2 - 5x + 4 &= 0 && \text{Original equation} \\ (x - 1)(x - 4) &= 0 && \text{Factor the polynomial.} \\ x - 1 = 0 \quad \text{or} \quad x - 4 &= 0 && \text{Zero Product Property} \\ x = 1 & \quad \quad \quad x = 4 && \end{aligned}$$

### PRACTICE PROBLEMS

Factor each polynomial.

46.  $w^2 + 4w$

47.  $n^2 + 8n + 15$

48.  $x^2 - 9x + 18$

49.  $3y^2 + 2y - 4$

Solve each equation by factoring.

50.  $10r^2 - 35r = 0$

51.  $x^2 + 13x + 36 = 0$

52.  $3b^2 - 15 = 4b$

53.  $2y^2 = 5y + 12$

54.  $-4b = -3b^2 + 15$

## TOPIC 9: QUADRATIC FORMULA

### EXAMPLE Two Rational Roots

**1** Solve  $x^2 - 12x = 28$  by using the Quadratic Formula.

First, write the equation in the form  $ax^2 + bx + c = 0$  and identify  $a$ ,  $b$ , and  $c$ .

$$ax^2 + bx + c = 0$$
$$x^2 - 12x = 28 \rightarrow 1x^2 - 12x - 28 = 0$$

Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-28)}}{2(1)} \quad \text{Replace } a \text{ with } 1, b \text{ with } -12, \text{ and } c \text{ with } -28.$$

$$= \frac{12 \pm \sqrt{144 + 112}}{2} \quad \text{Simplify.}$$

$$= \frac{12 \pm \sqrt{256}}{2} \quad \text{Simplify.}$$

$$= \frac{12 \pm 16}{2} \quad \sqrt{256} = 16$$

$$x = \frac{12 + 16}{2} \text{ or } x = \frac{12 - 16}{2} \quad \text{Write as two equations.}$$

$$= 14 \quad = -2 \quad \text{Simplify.}$$

The solutions are  $-2$  and  $14$ . Check by substituting each of these values into the original equation.

### PRACTICE PROBLEMS

Solve each equation using Quadratic Formula. Express answers in simplified radical form.

55.  $x^2 + 6x = 16$

56.  $2x^2 + 25x + 33 = 0$

57.  $3x^2 + 5x + 1 = 0$

58.  $x^2 - 8x = -9$

## TOPIC 10: SYSTEMS OF EQUATIONS

**Substitution** One algebraic method is the **substitution method**. Using this method, one equation is solved for one variable in terms of the other. Then, this expression is substituted for the variable in the other equation.

### EXAMPLE Solve by Using Substitution

1 Use substitution to solve the system of equations.

$$x + 2y = 8$$

$$\frac{1}{2}x - y = 18$$

Solve the first equation for  $x$  in terms of  $y$ .

$$x + 2y = 8 \quad \text{First equation}$$

$$x = 8 - 2y \quad \text{Subtract } 2y \text{ from each side.}$$

Substitute  $8 - 2y$  for  $x$  in the second equation and solve for  $y$ .

$$\frac{1}{2}x - y = 18 \quad \text{Second equation}$$

$$\frac{1}{2}(8 - 2y) - y = 18 \quad \text{Substitute } 8 - 2y \text{ for } x.$$

$$4 - y - y = 18 \quad \text{Distributive Property}$$

$$-2y = 14 \quad \text{Subtract 4 from each side.}$$

$$y = -7 \quad \text{Divide each side by } -2.$$

Now, substitute the value for  $y$  in either original equation and solve for  $x$ .

$$x + 2y = 8 \quad \text{First equation}$$

$$x + 2(-7) = 8 \quad \text{Replace } y \text{ with } -7.$$

$$x - 14 = 8 \quad \text{Simplify.}$$

$$x = 22$$

The solution of the system is  $(22, -7)$ .

### PRACTICE PROBLEMS

Solve by Substitution.

59.  $2x - 3y = 2$   
 $x + 2y = 15$

60.  $7y = 26 + 11x$   
 $x - 3y = 0$



**Elimination** Another algebraic method is the **elimination method**. Using this method, you eliminate one of the variables by adding or subtracting the equations. When you add two true equations, the result is a new equation that is also true.

### EXAMPLE Solve by Using Elimination

**3** Use the elimination method to solve the system of equations.

$$4a + 2b = 15$$

$$2a + 2b = 7$$

In each equation, the coefficient of  $b$  is 2. If one equation is subtracted from the other, the variable  $b$  will be eliminated.

$$\begin{array}{r} 4a + 2b = 15 \\ (-) 2a + 2b = 7 \\ \hline \end{array}$$

$$2a = 8 \quad \text{Subtract the equations.}$$

$$a = 4 \quad \text{Divide each side by 2.}$$

Now find  $b$  by substituting 4 for  $a$  in either original equation.

$$2a + 2b = 7 \quad \text{Second equation}$$

$$2(4) + 2b = 7 \quad \text{Replace } a \text{ with 4.}$$

$$8 + 2b = 7 \quad \text{Multiply.}$$

$$2b = -1 \quad \text{Subtract 8 from each side.}$$

$$b = -\frac{1}{2} \quad \text{Divide each side by 2.}$$

The solution is  $\left(4, -\frac{1}{2}\right)$ .

## PRACTICE PROBLEMS

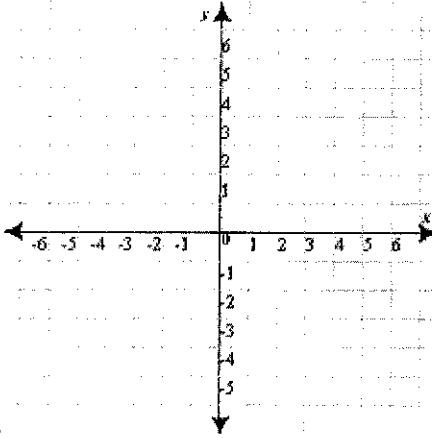
Solve each system of equations using elimination.

61.  $3x + 4y = 14$   
 $4x + 5y = 17$

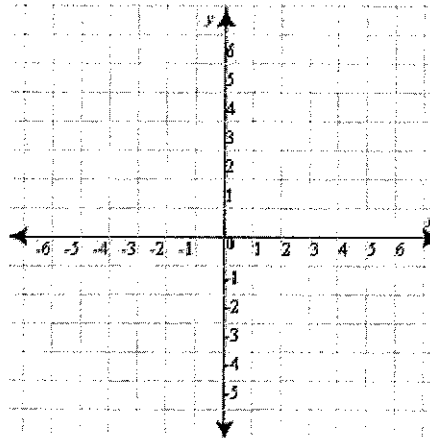
62.  $2x - 4y = 28$   
 $4x = 17 - 5y$

**TOPIC 1: GRAPHING ORDERED PAIRS**

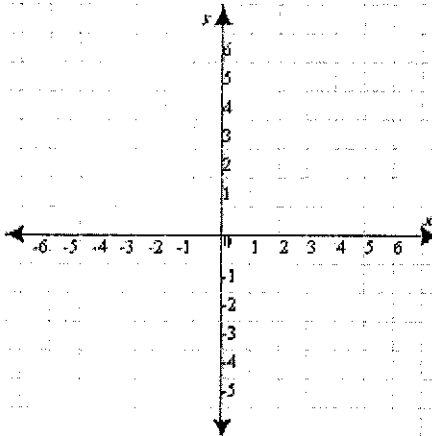
1.



2.



3.



**TOPIC 2: EVALUATING ALGEBRAIC EXPRESSIONS**

4. \_\_\_\_\_

5. \_\_\_\_\_

6. \_\_\_\_\_

7. \_\_\_\_\_

8. \_\_\_\_\_

**TOPIC 3: SOLVING EQUATIONS**

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11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

14. \_\_\_\_\_

15. \_\_\_\_\_

16. \_\_\_\_\_

**TOPIC 4: SOLVING INEQUALITIES**

17. \_\_\_\_\_

18. \_\_\_\_\_

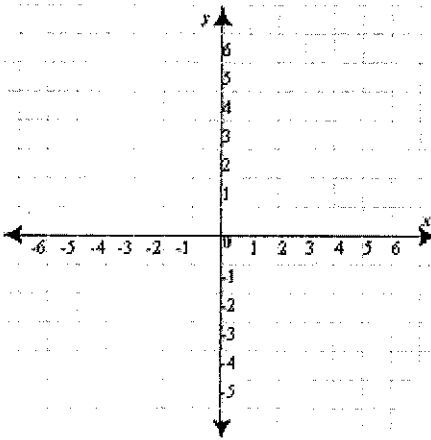
19. \_\_\_\_\_

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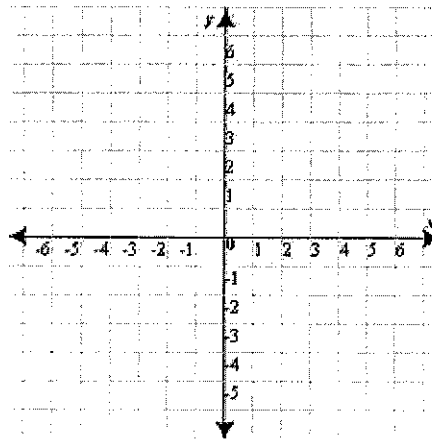
21. \_\_\_\_\_

**TOPIC 5: GRAPHING EQUATIONS**

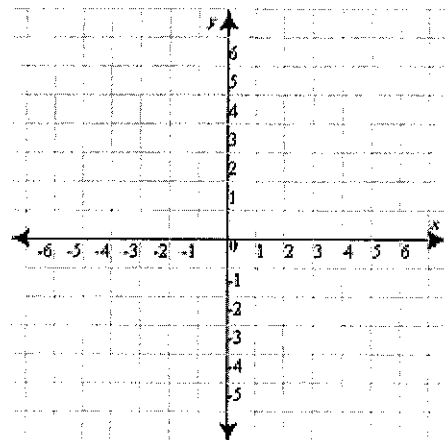
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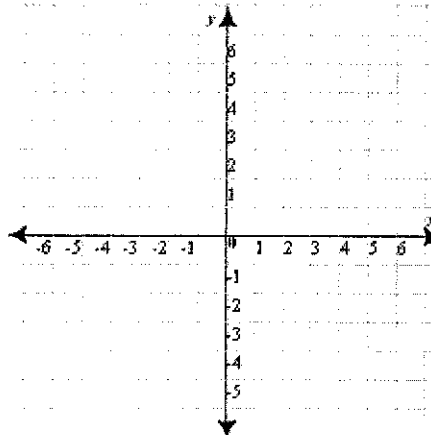
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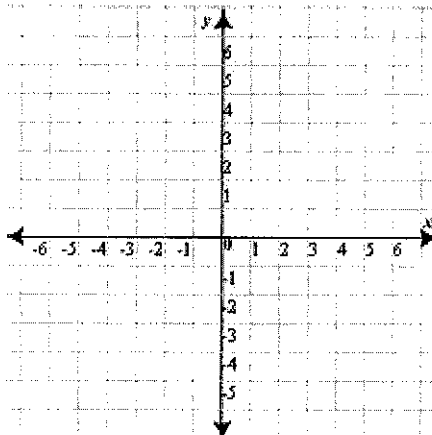
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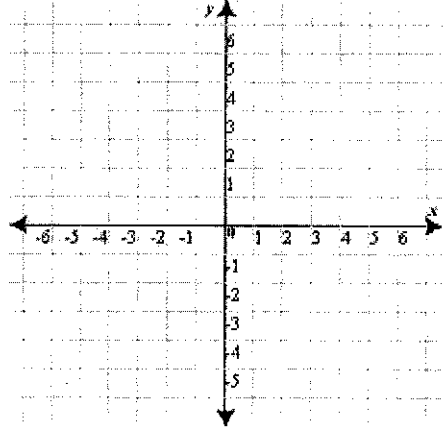
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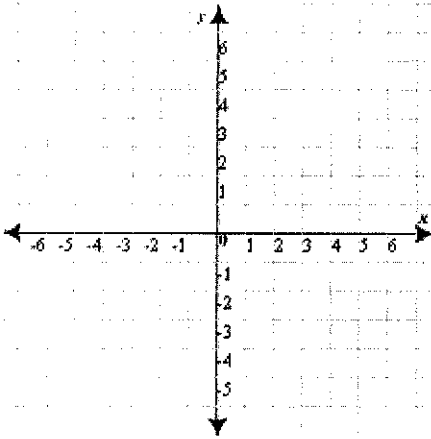
26.



27.



28.



**TOPIC 6: SQUARE ROOTS AND SIMPLIFYING RADICALS**

29. \_\_\_\_\_

30. \_\_\_\_\_

31. \_\_\_\_\_

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36. \_\_\_\_\_

**TOPIC 7: MULTIPLYING POLYNOMIALS**

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45. \_\_\_\_\_

**TOPIC 8: FACTORING TO SOLVE EQUATIONS**

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49. \_\_\_\_\_

50. \_\_\_\_\_

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53. \_\_\_\_\_

54. \_\_\_\_\_

**TOPIC 9: QUADRATIC FORMULA**

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56. \_\_\_\_\_

57. \_\_\_\_\_

58. \_\_\_\_\_

**TOPIC 10: SYSTEMS OF EQUATIONS**

59. \_\_\_\_\_

60. \_\_\_\_\_

61. \_\_\_\_\_

62. \_\_\_\_\_